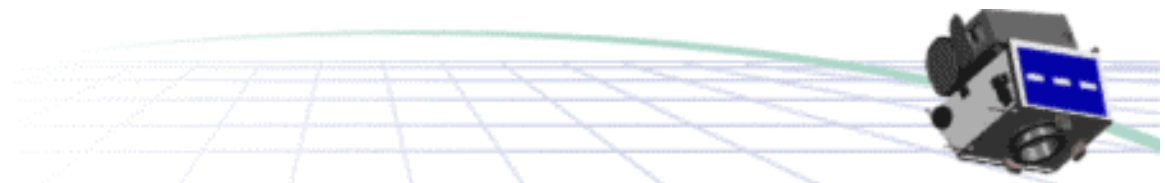


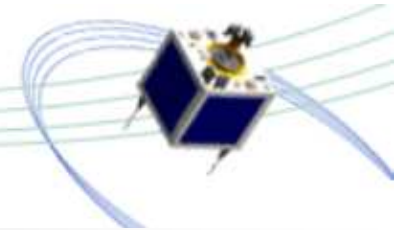
The SSC Control Systems Group's Air Bearing Table Facility

Jon Seddon and Congying Han

Supervisor: Dr. Alexandre Pechev

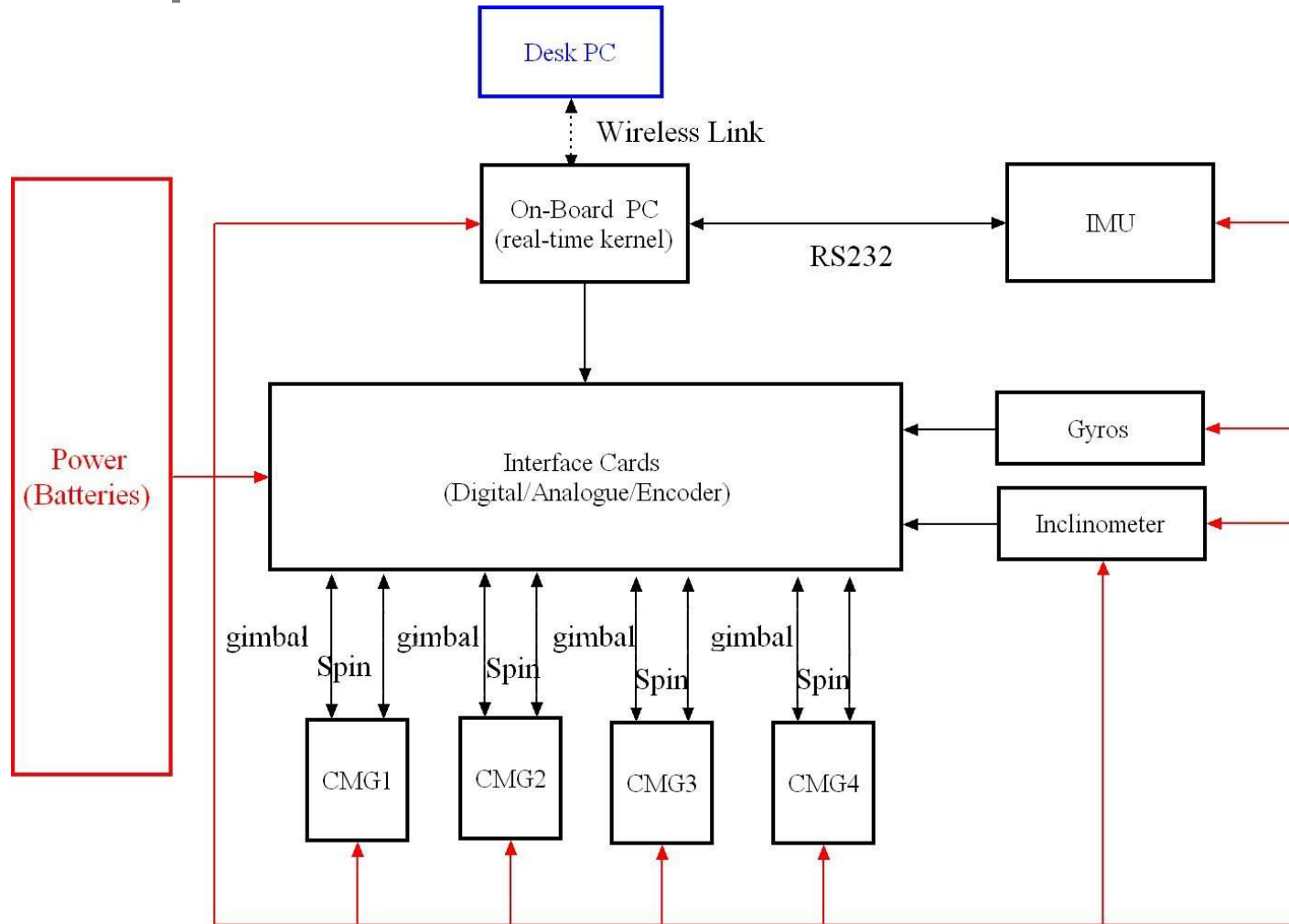
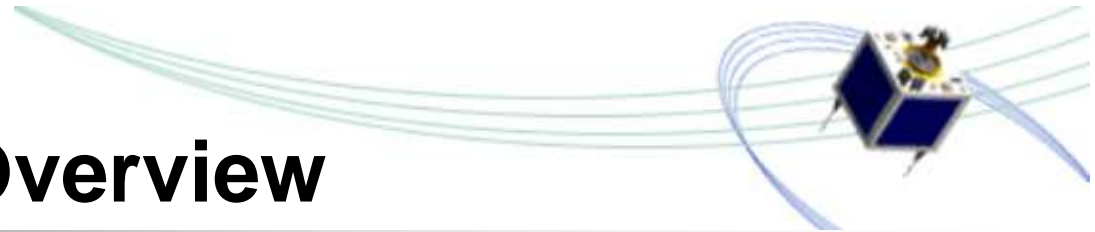
28th May 2008





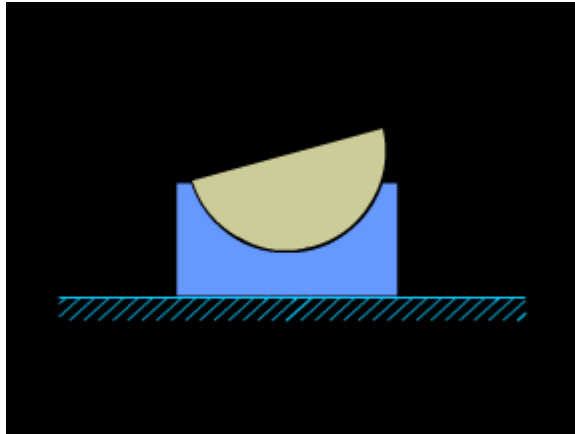
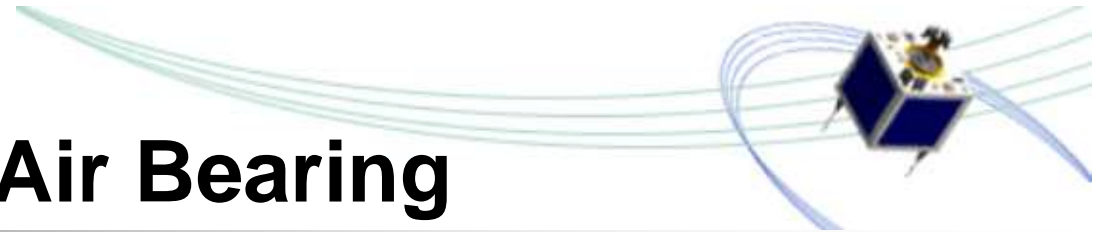
- Dave Fishlock from Technical Services
- Richard Reffell and the Mechanical Workshop team
- Tom Frame and Alex Domke

System Overview





- **Hardware Overview**
 - Air bearing, sensors, actuators, computing and levelling system
- **Theory**
 - Co-ordinate systems, attitude estimation, control, inertia matrix
- **Experimental Results**
 - Singularity avoidance, 3-axis control



- SRA 250 Spherical Air Bearing from Speciality Components, Inc.
- Tilts through $\pm 30^\circ$ in two axes and can support 230 kg from a 80 psi compressed air supply and $3\mu\text{m}$ air gap



- Microstrain 3DM-GX1 Inertial Measurement Unit – 3-axis gyro, accelerometer, compass and processor for filtered attitude angles and rates as euler angles or quaternions. 100 Hz bandwidth.

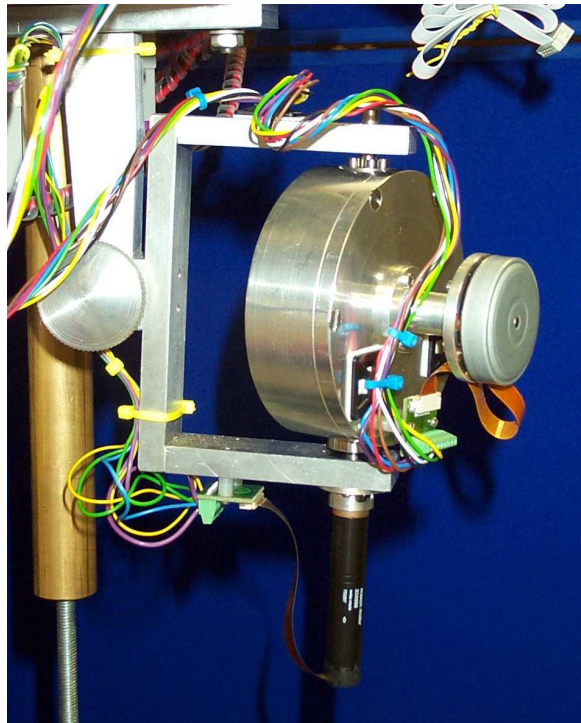
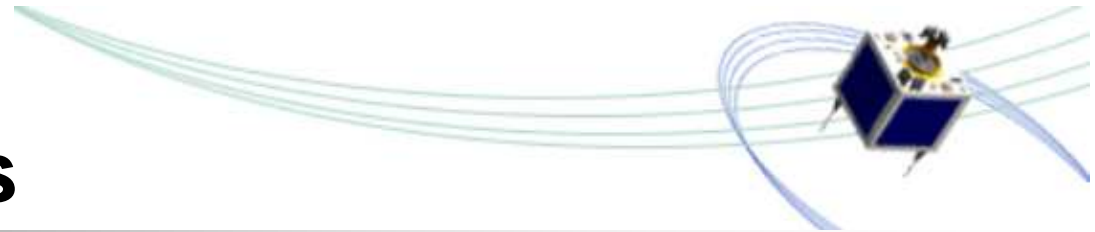




- Memsense TriRate $\pm 150^\circ/\text{s}$ MEMS Gyro – 3-axis rate data, but prone to drift. Estimator now being developed to overcome this. Bandwidth of 50 Hz.



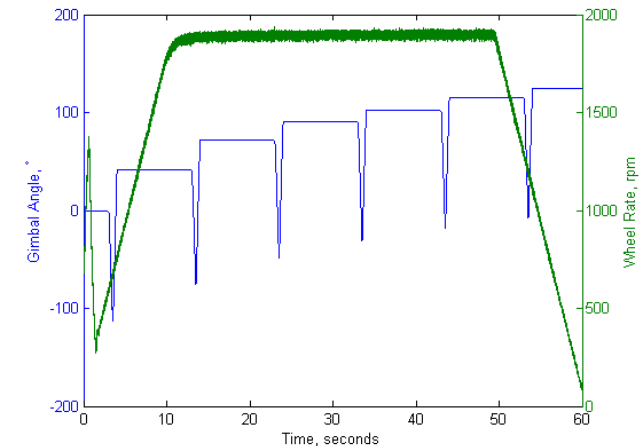
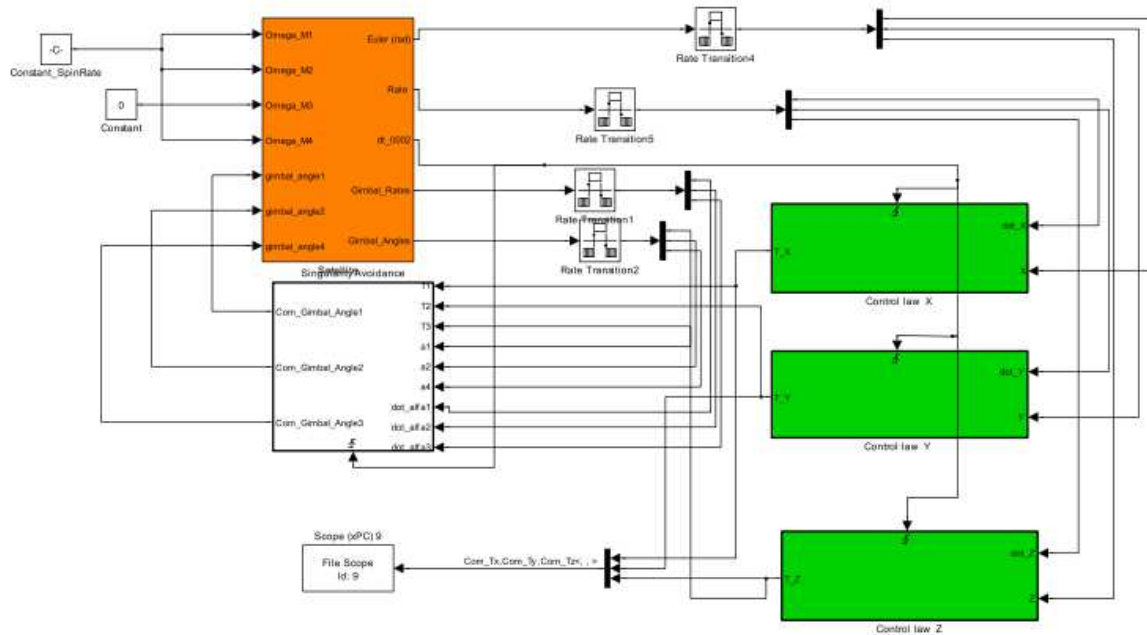
- Measurement Specialities
Accustar II Dual Axis Clinometer – 2-axis attitude angles. 0.25 Hz bandwidth.

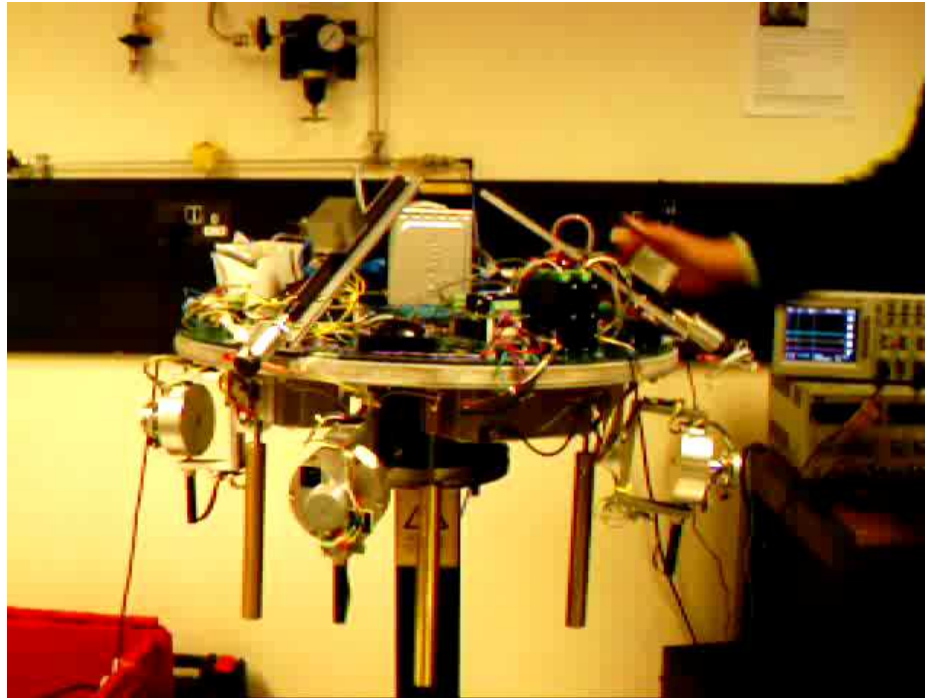
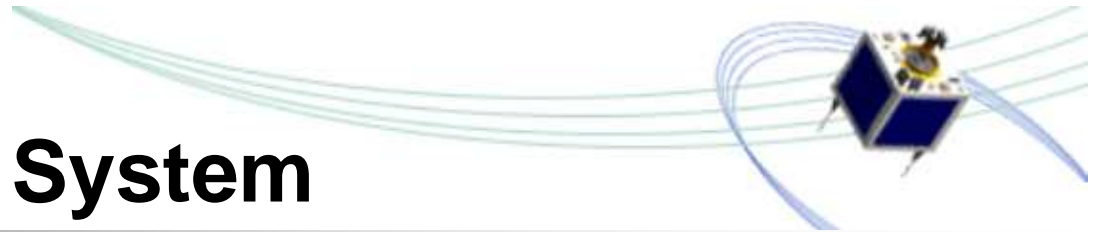


- 4 Control Moment Gyros designed and manufactured at SSC
- Gimbal rate of up to 4 rad/s gives 0.06 Nms angular momentum and 0.24 Nm torque output each as a CMG
- Variable alignment angle to build pyramid configurations
- Can also be used as reaction/momentum wheels with 0.07 Nm output. Is also a VSCMG

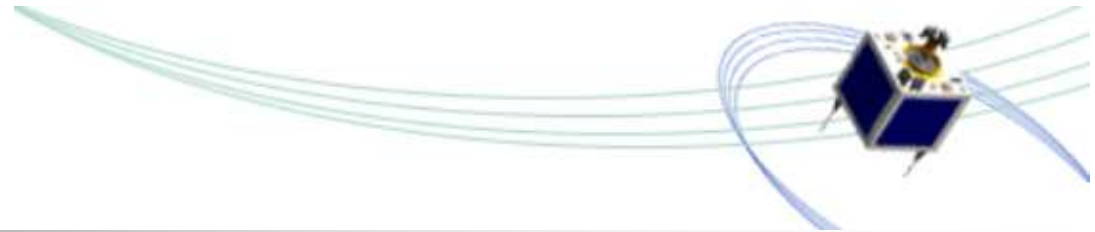


- Embedded PC running Simulink's xPC Target Real Time Operating System
- National Instruments I/O cards, digital and analogue I/O, PWM outputs, encoder inputs
- Wireless network link to host PC





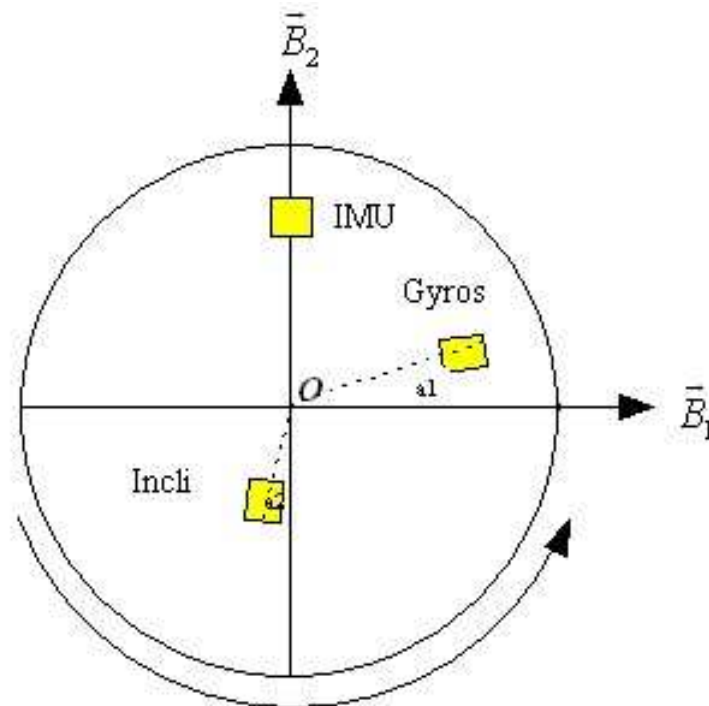
- 3 linear motors, with data from the IMU, automatically balance the table
- Can be used to generate disturbances



- Coordinates Transformation
- Attitude Estimation
- Control of Motors
- System Modelling
- Estimation of Inertia Matrix and Gravity Torque
- Experimental Results
- Singularity Avoidance
- 3-axis Control
- Future Work and Possible Activities



Coordinates



Transformation Matrix (Sensors)

(a1,a2 are estimated roughly)

- From gyros to body frame

$$R = \begin{bmatrix} -\cos(0.04) & \sin(0.04) & 0 \\ \sin(0.04) & \cos(0.04) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- From inclinometer to body frame

$$R = \begin{bmatrix} \sin(0.1367) & \cos(0.1367) & 0 \\ -\cos(0.1367) & \sin(0.1367) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

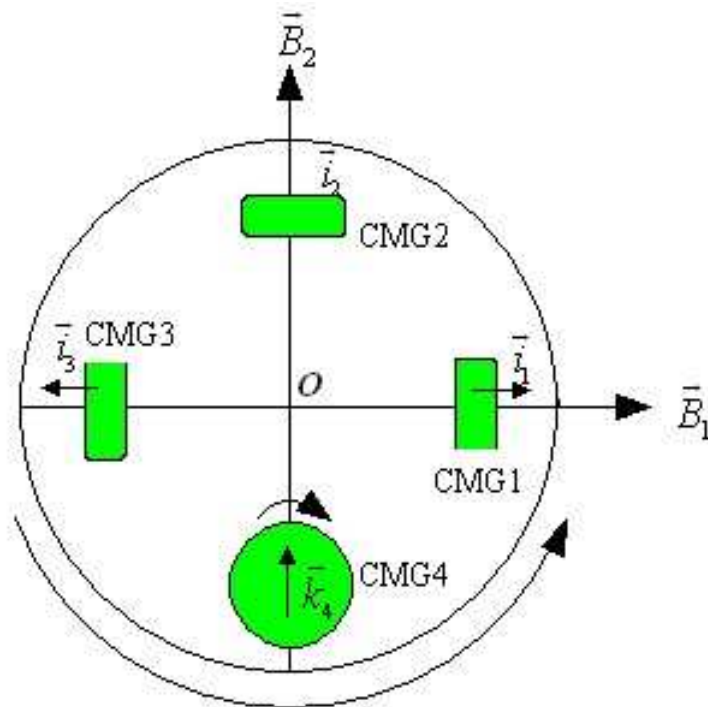
- From IMU to body frame

$$R = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



Transformation Matrix (Actuators)

Coordinates



- As RWs:

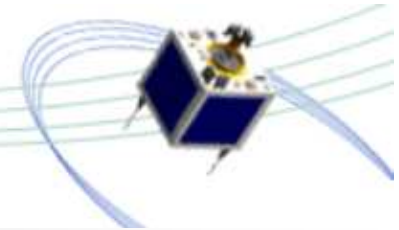
$$\vec{h}_1 = \vec{B}_1, \vec{h}_2 = \vec{B}_2, \vec{h}_3 = -\vec{B}_1, \vec{h}_4 = -\vec{B}_3$$

- As CMGs (1,2 and 4):

$$\vec{h}_1 = \cos(\alpha_1)\vec{B}_1 + \sin(\alpha_1)\vec{B}_2$$

$$\vec{h}_2 = -\sin(\alpha_2)\vec{B}_1 + \cos(\alpha_2)\vec{B}_2$$

$$\vec{h}_4 = \sin(\alpha_4)\vec{B}_1 - \cos(\alpha_4)\vec{B}_3$$



- Measurements from Gyros and Inclinator

Gyros:

Provide angular rates of three axes

Are sensitive with temperature

Remove constant bias first before experiments

Inclinator:

Provides Euler angles of x-,y-axis

- Measurements from IMU

Provide Euler rates, Euler angles and accelerations of

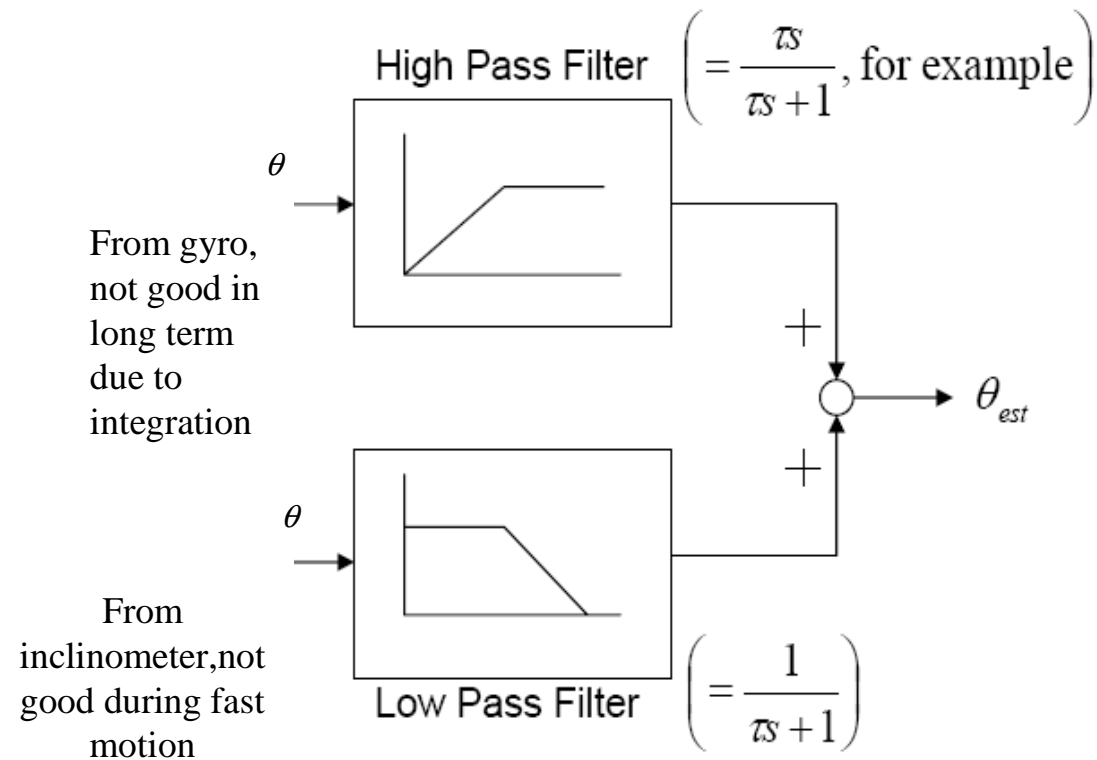
3 axes. Because of magnetic field caused by motors, Euler angle Z cannot be reliable

Integrated Kalman Filters



Complementary Filter

Two-axis attitude measured by Gyros and Inclinometer

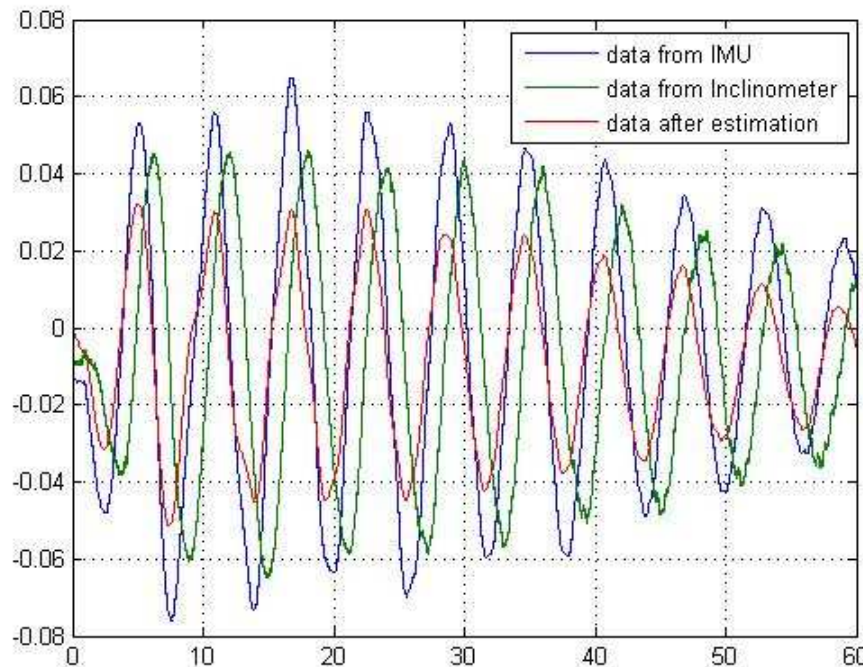


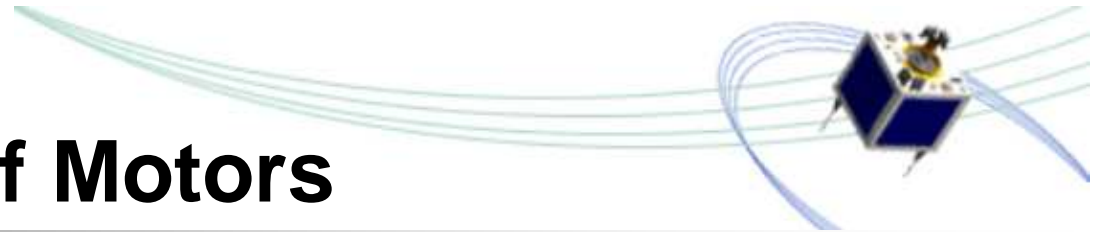


- Experiment Result

High pass filter = $\frac{0.56^2 s^2}{(0.56s+1)^2}$

Low pass filter = $\frac{2 \times 0.56s + 1}{(0.56s+1)^2}$

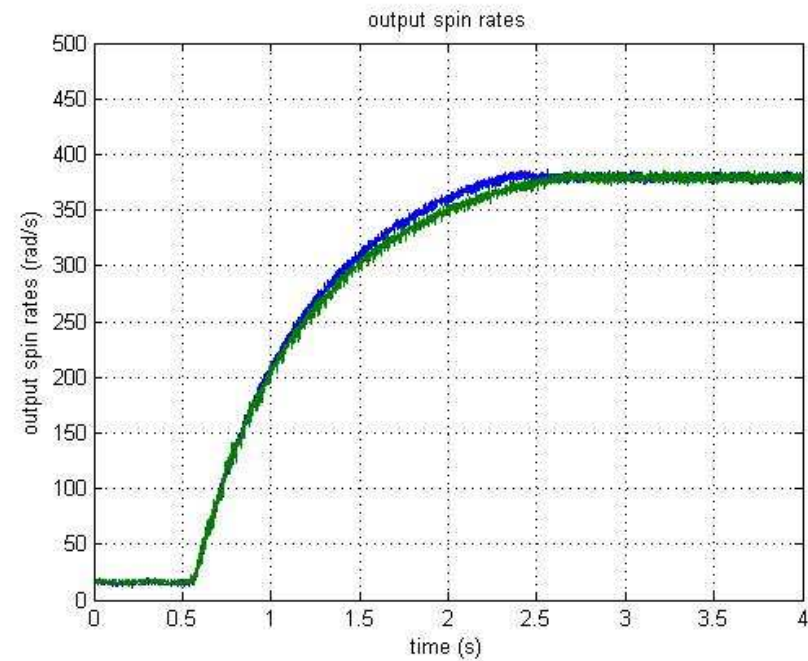
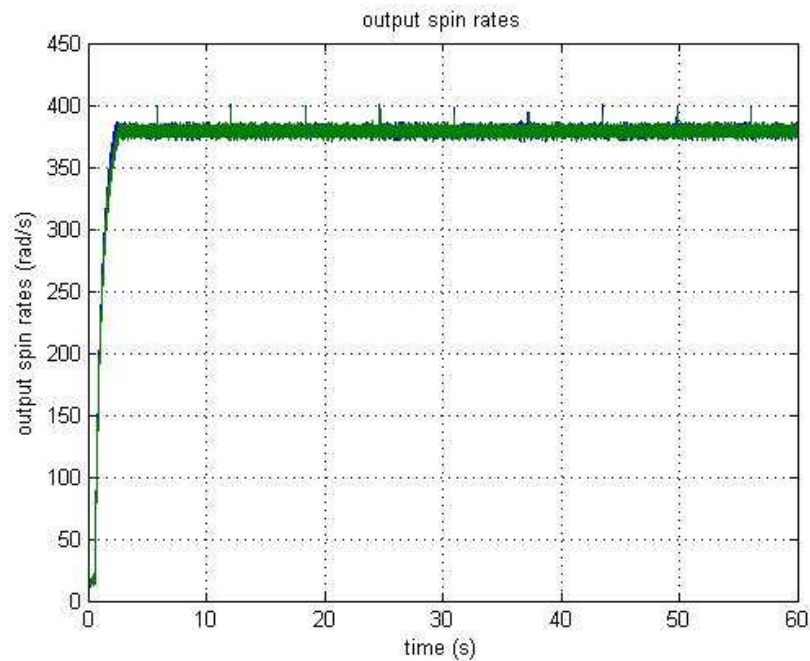




- Control of Spinning Speed

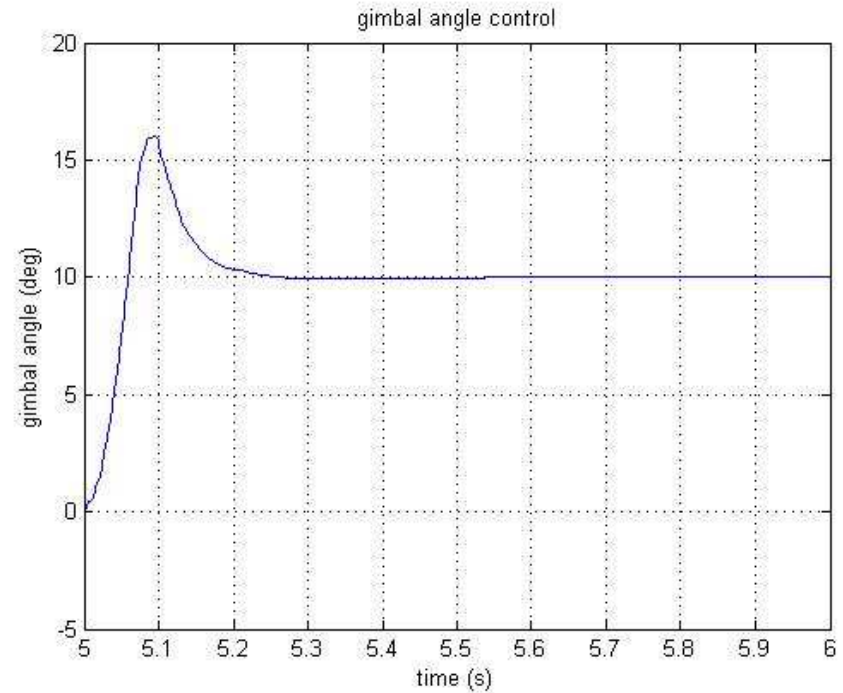
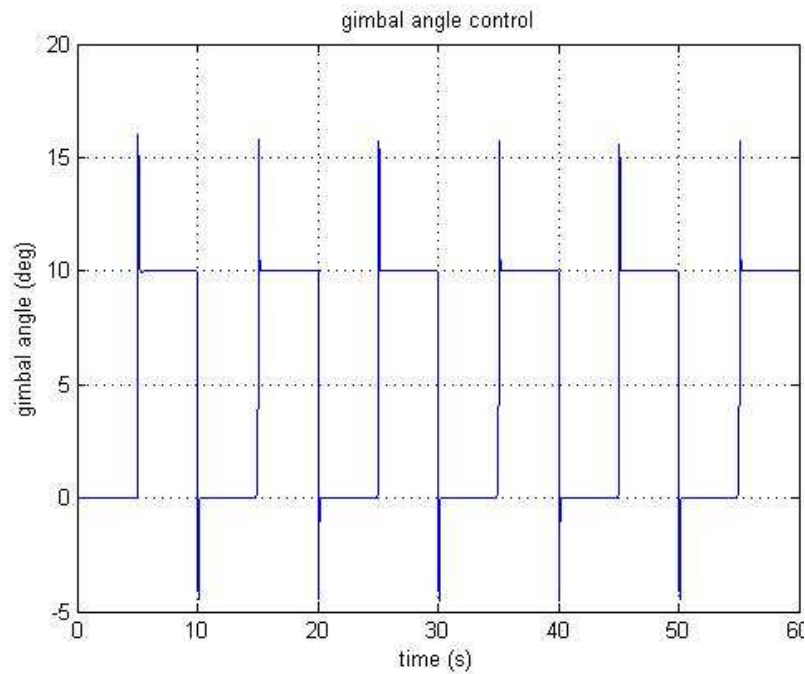
Open-loop control

Spinning speed dumping after 400rad/s (max speed 380rad/s)





- Control of Gimbal Angles
Current results on close-loop control





- Modelling of Gravity Torque: $T_{mg} = \vec{r} \times mg\vec{K}$

Body Frame

$$T_{mg} = mg \begin{bmatrix} r_y \cos \phi \cos \theta - r_z \sin \phi \cos \theta \\ -r_x \cos \phi \cos \theta - r_z \sin \theta \\ r_x \cos \phi \cos \theta + r_y \sin \theta \end{bmatrix}$$

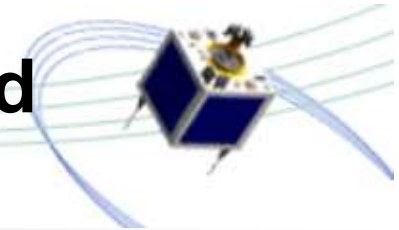
- Air-bearing Table Model

$$I\dot{\omega} + \dot{h} + \omega \times (I\omega + h) = T_{mg} + T_d$$

$$I_1 \dot{\omega}_1 = -\dot{h}_1 - (I_3 - I_2)\omega_2\omega_3 + h_2\omega_3 - h_4\omega_2 + T_{mg1} + T_{d1}$$

$$I_2 \dot{\omega}_2 = -\dot{h}_2 - (I_1 - I_3)\omega_1\omega_3 - h_1\omega_3 + h_4\omega_1 + T_{mg2} + T_{d2}$$

$$I_3 \dot{\omega}_3 = -\dot{h}_3 - (I_2 - I_1)\omega_1\omega_2 - h_2\omega_1 + h_1\omega_2 + T_{mg3} + T_{d3}$$



- Initial Estimation of Inertia Matrix

The moment of inertia depends upon how an object's mass is distributed relative to its pivot point

----Look at all the components attached to the platform as rigid spheres

----Using parallel-axis theory

Sum of MOI of each component = MOI of air-bearing table

For a simplified representation we get:

$$I_1 = 0.5899$$

$$I_2 = 0.5933$$

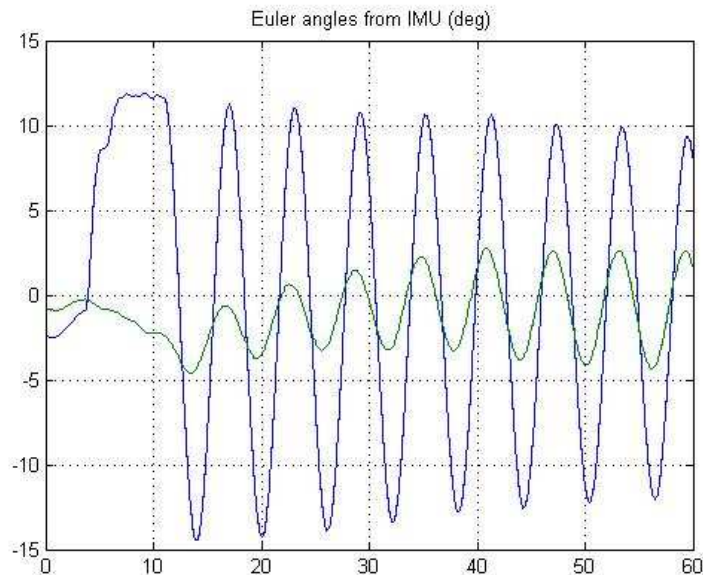
$$I_3 = 1.1093$$



- Initial Estimation of Gravity Torque

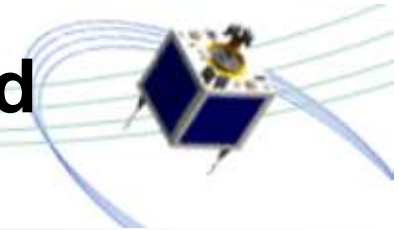
Since actuators/sensors are mounted symmetrically, $r_x = r_y = 0$

Look air-bearing table as pendulum, and give air-bearing table an initial attitude



$$\frac{2\pi}{6} = \sqrt{\frac{mgr_z}{I_1}}$$

$$mgr_z = 0.6469 \text{ (Nm)}$$



- Recursive Least-square Estimation

$$\bar{A}_k x = \bar{T}_k$$

$$\begin{bmatrix} \dot{\omega}_1 & -\omega_2\omega_3 & \omega_2\omega_3 & 0 & -\cos\phi\cos\theta & \sin\phi\cos\theta \\ \omega_1\omega_3 & \dot{\omega}_2 & -\omega_1\omega_3 & \cos\phi\cos\theta & 0 & \sin\theta \\ -\omega_1\omega_2 & \omega_1\omega_2 & \dot{\omega}_3 & -\cos\phi\cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ mgr_x \\ mgr_y \\ mgr_z \end{bmatrix} = I_w \begin{bmatrix} -\dot{\omega}_{w1} + \omega_{w2}\omega_3 - \omega_{w3}\omega_2 \\ -\dot{\omega}_{w2} - \omega_{w1}\omega_3 + \omega_{w3}\omega_1 \\ -\dot{\omega}_{w3} + \omega_{w1}\omega_2 - \omega_{w2}\omega_1 \end{bmatrix}$$

- Minimizes $\|\bar{A}_k x - \bar{T}_k\|$

$$\hat{x}_k = \hat{x}_{k-1} + L_k (\bar{T}_k - \bar{A}_k \hat{x}_{k-1})$$

$$L_k = P_k \bar{A}_k$$

$$P_k = (I - L_k \bar{A}_k) P_{k-1}$$



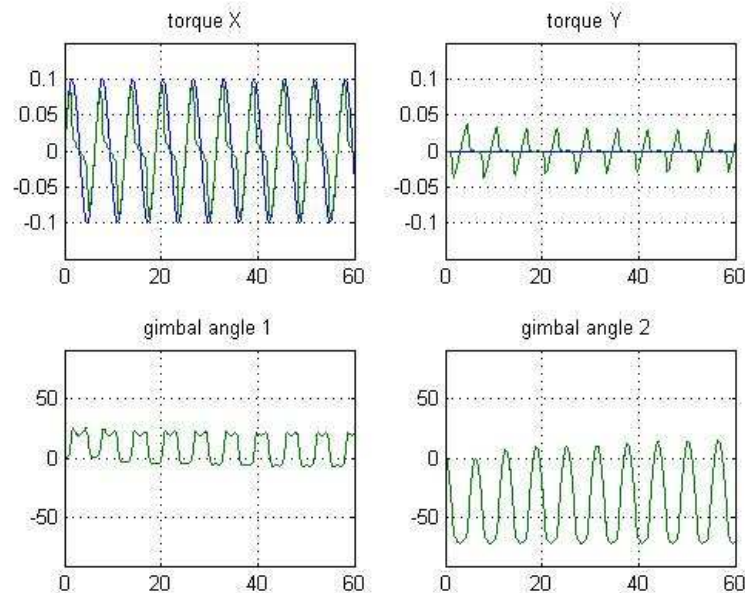
- Singularity Avoidance (SR)

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = h_0 \begin{bmatrix} -\sin \alpha_1 & -\cos \alpha_2 \\ \cos \alpha_1 & -\sin \alpha_2 \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix}$$

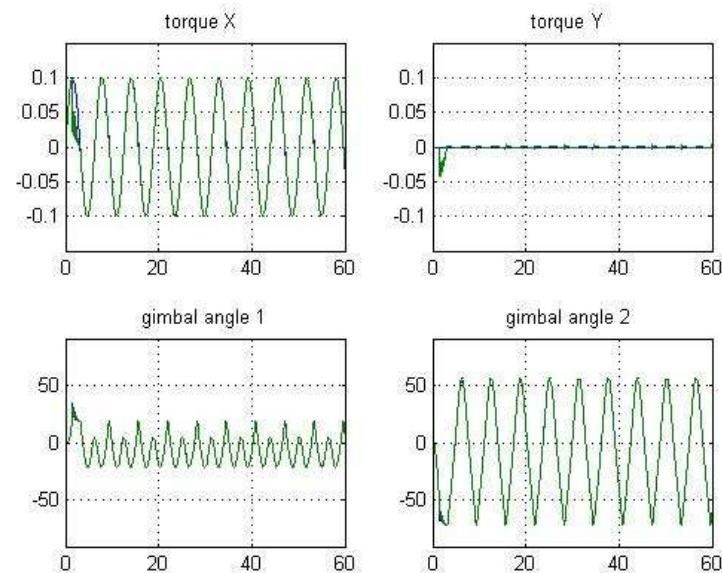
$$\dot{\alpha} = J(\alpha)^T (J(\alpha)J^T(\alpha) + \lambda I)^{-1} T$$

$$\lambda = \lambda_0 e^{-\det(JJ^T)}$$

$\lambda_0 = 0.1$

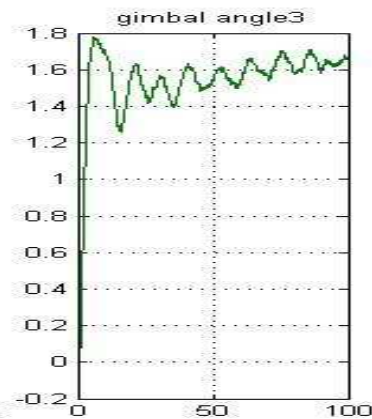
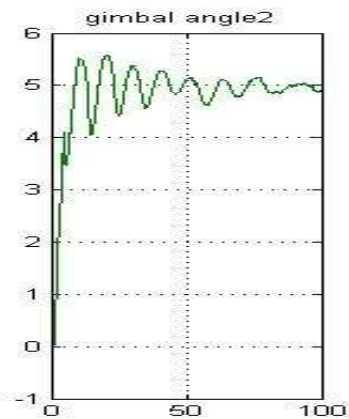
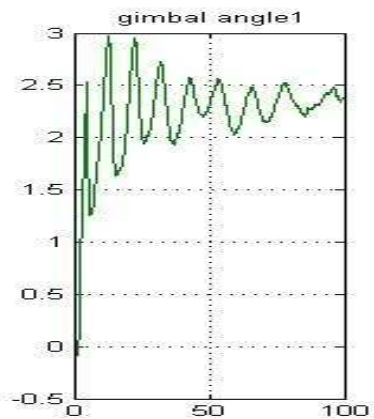
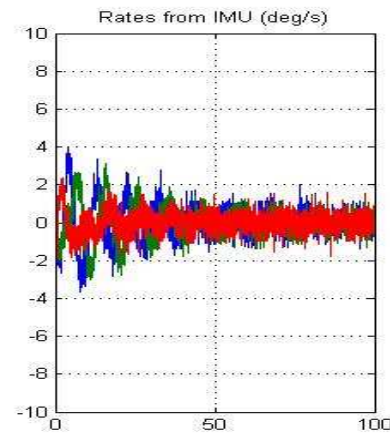
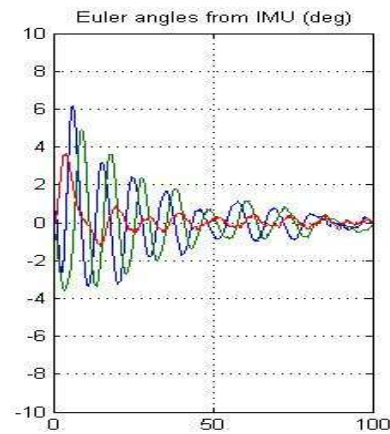


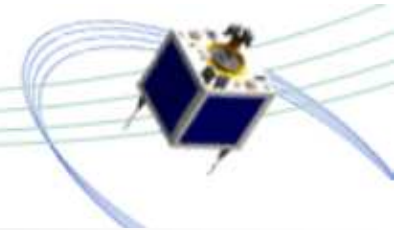
$\lambda_0 = 0.001$





- 3-Axis Control with 3 CMGs (CMG-1,-2 and -4)





- Attitude estimation using EKF
- Estimation of inertia matrix and gravity torque (by Dr Horri)
- Test of new singular avoidance proposed by Dr Pechev
- Test of new actuators/sensors
- New control/filter algorithms
- Tracking algorithms
- Evaluation of my underactuated control
- Robotics/formation flying by incorporating other facilities

Any Questions?

